DETERMINATION OF A SET OF MATERIAL THERMOPHYSICAL CHARACTERISTICS FROM DATA OF NONSTATIONARY MEASUREMENTS DURING HEATING BY LOCAL HEAT SOURCES

A. S. Golosov, V. I. Zhuk, A. A. Lopashev, and D. N. Chubarov

UDC 536.2.08

Exact explicit dependences connecting material thermophysical characteristics to the results of measuring nonstationary values of the primary parameters during specimen heating by local thermal sources with power varying arbitrarily in time are presented.

Heating specimens of materials being investigated by local heat sources of different configurations [1-3] is used extensively in thermophysical experiment practice. It should be noted that the computed dependences for determination of the thermophysical characteristics (TPC) are approximate in nature in the majority of cases and, moreover, are obtained for particular laws of the variation of the supplied thermal fluxes in time. A solution of the problem of determining a TPC set by using measurement data of nonstationary temperatures and thermal fluxes during heating of semi-bounded specimens by local surface heat sources of variable power is presented in this paper as a development of the approach elucidated in [4].

We examine the following mathematical model in application to a two-dimensional heat propagation process in a material

$$\frac{\partial^2 T(r, z, \tau)}{\partial r^2} + \frac{1}{r}\frac{\partial T(r, z, \tau)}{\partial r} + \frac{\partial^2 T(r, z, \tau)}{\partial z^2} = \frac{1}{a}\frac{\partial T(r, z, \tau)}{\partial \tau}, \tag{1}$$

$$-\lambda \left.\frac{\partial T(r, z, \tau)}{\partial z}\right|_{z=0} = q(r, \tau), \tag{2}$$

$$T(r, z, \tau)_{r\to\infty} = T(r, z, \tau)_{z\to\infty} = 0, \tag{2'}$$

$$T(r, z, 0) = 0, \tag{2''}$$

that holds, say, during heating of a half-space by local sources forming temperature fields with axial symmetry.

By using Laplace integral $T(r, z, s) = \int_0^\infty \exp(-s\tau)\, T(r, z, \tau)d\tau$ and Hankel $T(p,z,s) = \int_0^\infty rJ_0(pr)\, T(r, z, s)dr$ transforms, the solution of the system (1)-(2'') can be represented in the form

$$T(r, z, s) = \frac{1}{\lambda}\int_0^\infty \frac{pJ_0(pr)\, q(p, s)\exp\left(-z\sqrt{p^2+\frac{s}{a}}\right)}{\left(p^2+\frac{s}{a}\right)^{1/2}}\, dp, \tag{3}$$

where $q(p, s) = \int_0^\infty rJ_0(pr)q(r, s)dr$. For heat sources concentrated at a point or on a circle of radius R_0, or distributed uniformly within the limits of a circle of radius R_0, respectively, the following representations hold for $g(p, s)$: $\frac{Q(s)}{2\pi}$, $\frac{Q(s)}{2\pi} J_0(pR_0)$, $\frac{Q(s)}{\pi R_0 p} J_1(pR_0)$, $Q(s)$ is the power of the heat source.

Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 60, No. 2, pp. 327-335, February, 1991. Original article submitted November 4, 1989.

In the case of a source concentrated on a circle or distributed uniformly within the limits of a circle, we have the following expressions for the temperatures at r = 0 and z = 0

$$T(0,\ 0,\ s)=\frac{Q(s)}{2\pi R_0\lambda}\exp\left(-\sqrt{\frac{s}{a}}\,R_0\right), \tag{4}$$

$$T(0,\ 0,\ s)=\frac{Q(s)}{\pi R_0^2\sqrt{\lambda c\rho}\sqrt{s}}\left[1-\exp\left(-\sqrt{\frac{s}{a}}\,R_0\right)\right]. \tag{5}$$

In the case of a point source, we have at a distance R_0 away for z = 0

$$T(R_0,\ 0,\ s)=\frac{Q(s)}{2\pi R_0\lambda}\exp\left(-\sqrt{\frac{s}{a}}\,R_0\right). \tag{6}$$

For a source with a normal density distribution law over the surface $q(r,\ \tau)=q_0(\tau)$ $\exp(-\gamma r^2)$, which corresponds to $q(p,\ s)=q_0(s)\ \frac{1}{2\gamma}\exp\left(-\frac{p^2}{4\gamma}\right)$, we have for z = 0, r = 0 according to [5]

$$T(0,\ 0,\ s)=\frac{q_0(s)}{2\lambda}\frac{\sqrt{\pi}}{\sqrt{\gamma}}\exp\left(\frac{s}{4a\gamma}\right)\operatorname{erfc}\left(\frac{\sqrt{s}}{2\sqrt{a\gamma}}\right). \tag{7}$$

It follows from (4) and (5) that in the case of sources concentrated in a circle or at a point

$$\frac{d}{ds}\left[\overline{T}(s)/Q(s)\right]=-\frac{R_0}{2\sqrt{a}\sqrt{s}}\ \frac{T(s)}{Q(s)}, \tag{8}$$

where $\overline{T}(s)$ is the temperature at the center of the heating zone by a source concentrated on a circle of radius R_0 (r = 0, z = 0) or the temperature at a distance R_0 from a point source. There follows from (8)

$$\overline{T}'(s)\,Q(s)-\overline{T}(s)\,Q'(s)=-\frac{R_0}{2\sqrt{a}}\,\overline{T}(s)\,Q(s)\,s^{-1/2}, \tag{9}$$

from which according to [6] we have in the space of originals

$$a=\frac{F_1^2(\tau)}{F_2^2(\tau)}, \tag{10}$$

where

$$F_1(\tau)=\frac{R_0}{2\sqrt{\pi}}\int\limits_0^{\tau}\overline{T}(\tau-\theta)\int\limits_0^{\theta}(\theta-\vartheta)^{-1/2}Q(\vartheta)\,d\vartheta d\theta; \tag{10'}$$

$$F_2(\tau)=\int\limits_0^{\tau}(\tau-2\theta)\,\overline{T}(\tau-\theta)\,Q(\theta)\,d\theta. \tag{10''}$$

After having determined the parameter a by using the dependence (10), we find the parameter λ on the basis of (4) and (6) with the appropriate transform inversion procedures taken into account

$$\lambda=\frac{1}{4\pi^{3/2}\sqrt{a}}\int\limits_0^{\tau}Q(\tau-\theta)\,\theta^{-3/2}\exp\left(-\frac{R_0^2}{4a\theta}\right)d\theta/\overline{T}(\tau). \tag{11}$$

Formulas (10) and (11) permit determination of a set of the desired parameters a and λ, however the estimates obtained for the mentioned parameters will visibly be interdependent. On the other hand, estimates of the parameters a and λ can be obtained by independent means by using measurement data in two realizations with different dimensions of the sources concentrated on a circle or by using results of temperature measurements at different distances from point sources. If one of the sources concentrated in the circle has a radius R_0, or in the case of point sources (in one realization the temeprature measurements were performed at a distance R_0 from the source, and at the distance $2R_0$ in the other), then there follows from (4) and (6)

$$\left[\frac{\bar{T}_1(s)}{Q_1(s)}\right]^2 (2\pi R_0\lambda)^2 = \frac{\bar{T}_2(s)}{Q_2(s)} (4\pi R_0\lambda), \tag{12}$$

from which

$$\lambda \bar{T}_1^2(s) Q_2(s) = \frac{1}{\pi R_0} \bar{T}_2(s) Q_1^2(s), \tag{13}$$

where $\bar{T}_1$, $\bar{T}_2$ are the temperatures at the center of the heating zone in realizations with sources concentrated in circles of radii R_0 and $2R_0$ (or temperatures at the distances R_0 and $2R_0$ from point sources), Q_1 and Q_2 are powers of the sources in the first and second realizations. According to [6] we have in the space of originals

$$\lambda = \frac{\Psi_1(\tau)}{\Psi_2(\tau)}, \tag{14}$$

where

$$\Psi_1(\tau) = \frac{1}{\pi R_0} \int_0^\tau Q_1(\tau-\theta) \int_0^\theta \bar{T}_2(\theta-\vartheta) Q_1(\vartheta)\, d\vartheta d\theta, \tag{14'}$$

$$\Psi_2(\tau) = \int_0^\tau Q_2(\tau-\theta) \int_0^\theta \bar{T}_1(\theta-\vartheta) \bar{T}_1(\vartheta)\, d\vartheta d\theta. \tag{14''}$$

In this case the parameter a can be found on the basis of the dependence (10) by using measurement data in some realization. Moreover, the estimate of the parameter a can be obtained on the basis of the relationship

$$\frac{\bar{T}_1(s) Q_2(s)}{\bar{T}_2(s) Q_1(s)} = \exp\left(-\sqrt{\frac{s}{a}}\, R_0\right), \tag{15}$$

from which we write after differentiating with respect to the parameter s and going over to originals

$$a = \frac{\bar{F}_1^2(\tau)}{\bar{F}_2^2(\tau)}, \tag{16}$$

where

$$\bar{F}_1(\tau) = \frac{R_0}{2\sqrt{\pi}} \int_0^\tau \varphi_1(\tau-\theta) \int_0^\theta \varphi_2(\vartheta)(\theta-\vartheta)^{-1/2} d\vartheta d\theta; \tag{16'}$$

$$\bar{F}_2(\tau) = \int_0^\tau (2\theta-\tau)\varphi_1(\tau-\theta)\varphi_2(\theta)\, d\theta; \tag{16''}$$

$$\varphi_1(\theta) = \int_0^\theta \bar{T}_1(\theta-\vartheta) Q_2(\vartheta)\, d\vartheta, \quad \varphi_2(\theta) = \int_0^\theta \bar{T}_2(\theta-\vartheta) Q_1(\vartheta)\, d\vartheta. \tag{16'''}$$

In the case of a source distributed within the limits of a circle of radius R_0 we have on the basis of (5)

$$\frac{d}{ds}\left[\frac{\bar{T}(s)}{\tilde{q}(s)}\right] = -\frac{R_0}{2\sqrt{a}} \frac{\bar{T}(s)}{q(s)} + \frac{R_0}{2\sqrt{s}\lambda}, \tag{17}$$

where $\tilde{q}(s) = q(s)/\sqrt{s}$, $\bar{T}(s) = T(0, 0, s)$, $q(s) = Q(s)/\pi R_0^2$. There follows from (17)

$$\frac{2\sqrt{a}}{R_0} [\bar{T}'(s)\tilde{q}(s) - \bar{T}(s)\tilde{q}'(s)] = (\lambda c\rho)^{-1/2} s^{-3/2} q^2(s) - s^{-1}\bar{T}(s) q(s). \tag{18}$$

After going over to originals, we obtain

$$\frac{\sqrt{a}}{R_0}\bar{\varphi}_1(\tau) + (\lambda c\rho)^{-1/2}\bar{\varphi}_2(\tau) = \bar{\varphi}_3(\tau), \tag{19}$$

where

$$\bar{\varphi}_1(\tau) = \int_0^\tau (\tau - 2\theta)\bar{T}(\tau - \theta)\tilde{q}(\theta)\,d\theta; \tag{19'}$$

$$\bar{\varphi}_2(\tau) = \frac{1}{\sqrt{\pi}}\int_0^\tau q(\tau - \theta)\int_0^\theta q(\vartheta)(\theta - \vartheta)^{1/2}d\vartheta d\theta; \tag{19''}$$

$$\bar{\varphi}_3(\tau) = \frac{1}{2}\int_0^\tau \bar{T}(\tau - \theta)\int_0^\theta q(\vartheta)\,d\vartheta d\theta, \quad \tilde{q}(\theta) = \frac{1}{\sqrt{\pi}}\int_0^\theta q(\vartheta)(\theta - \vartheta)^{-1/2}d\vartheta. \tag{19'''}$$

Since the parameters a, λ, $\lambda c\rho$ are assumed constant, then for known $\bar{T}(\tau)$, $q(\tau)$ the relation (19) permits determination of the parameters a, $\lambda c\rho$ on the basis of evaluating $\bar{\varphi}_k$ ($k = 1, 3$) for different time intervals of one realization or arbitrary time intervals of two realizations with different $Q(s)$ in each. Then

$$\frac{\sqrt{a}}{R_0} = \bar{\varphi}_{3i}\bar{\varphi}_{1i}^{-1}\,[1 - (\bar{\varphi}_{3j}/\bar{\varphi}_{3i})(\bar{\varphi}_{2i}/\bar{\varphi}_{2j})][1 - (\bar{\varphi}_{1j}/\bar{\varphi}_{1i})(\bar{\varphi}_{2i}/\bar{\varphi}_{2j})]^{-1}, \tag{20}$$

$$(\lambda c\rho)^{1/2} = \bar{\varphi}_{2i}\bar{\varphi}_{3i}^{-1}\,[1 - (\bar{\varphi}_{2j}/\bar{\varphi}_{2i})(\bar{\varphi}_{1i}/\bar{\varphi}_{1j})][1 - (\bar{\varphi}_{3j}/\bar{\varphi}_{3i})(\bar{\varphi}_{1i}/\bar{\varphi}_{1j})]^{-1}, \tag{21}$$

where the subscripts i, j refer to values of $\bar{\varphi}_1$, $\bar{\varphi}_2$, $\bar{\varphi}_3$, evaluated for different time intervals τ_i, τ_j of one realization or to arbitrary time intervals of two realizations with different $Q(S)$.

Estimates of the parameters a, $\lambda c\rho$ obtained by using (2) and (21) are visibly dependent since they are determined by starting from one functional relationship (19). Independent estimates of the parameters a and $\lambda c\rho$ can be obtained in this case by using measurement results in two realizations with the source dimensions R_0 and $2R_0$. It follows from (5) that a functional relation of the form

$$[1 - \sqrt{s}\sqrt{\lambda c\rho}\,\bar{T}_1(s)/q_1(s)]^2 = 1 - \sqrt{s}\sqrt{\lambda c\rho}\,\bar{T}_2(s)/q_2(s), \tag{22}$$

is valid for the mentioned source dimensions, where $\bar{T}$, q_1 refer to a realization with source dimensions R_0, and $\bar{T}_2$, q_2 to a realization with the source dimensions $2R_0$, $q_1(s) = Q_1(s)/(\pi R_0{}^2)$, $q_2(s) = Q_2(s)/(4\pi R_0{}^2)$.

It follows from (22)

$$\sqrt{\lambda c\rho}\,\bar{T}_1^2(s)\,q_2(s)\,s = \sqrt{s}\,[2\bar{T}_1(s)\,q_1(s)\,q_2(s) - \bar{T}_2(s)\,q_1^2(s)], \tag{23}$$

from which we obtain after going over to originals

$$(\lambda c\rho)^{1/2} = \frac{\tilde{\Psi}_1(\tau)}{\tilde{\Psi}_2(\tau)}, \tag{24}$$

where

$$\tilde{\Psi}_1(\tau) = \int_0^\tau \tilde{q}_1(\tau - \theta)\int_0^\theta [2\bar{T}_1(\vartheta)\,\tilde{q}_2(\theta - \vartheta) - \bar{T}_2(\vartheta)\,\tilde{q}_1(\theta - \vartheta)]\,d\vartheta d\theta; \tag{24'}$$

$$\tilde{\Psi}_2(\tau) = \int_0^\tau \bar{T}_1(\tau - \theta)\int_0^\theta \bar{T}_1(\vartheta)\,\tilde{q}_2(\theta - \vartheta)\,d\vartheta d\theta; \tag{24''}$$

$$\tilde{q}_1(\theta) = \frac{1}{\sqrt{\pi}}\int_0^\theta q_1(\vartheta)(\theta - \vartheta)^{-1/2}d\vartheta, \quad \tilde{q}_2(\theta) = \frac{1}{\sqrt{\pi}}\int_0^\theta q_2(\vartheta)(\theta - \vartheta)^{-1/2}d\vartheta. \tag{24'''}$$

In its turn the parameter a can be found by starting from the relation (5) with the source dimensions used taken into account:

$$\frac{\bar{T}_2(s)\,q_1(s)}{\bar{T}_1(s)\,q_2(s)} = 1 + \exp\left(-\sqrt{\frac{s}{a}}\,R_0\right), \tag{25}$$

from which we have by differentiating with respect to the parameter s

$$f_1'(s)f_2(s)-f_1(s)f_2'(s)=\frac{R_0}{2\sqrt{as}}[f_2^2(s)-f_1(s)f_2(s)], \tag{26}$$

where $f_1(s) = \overline{T}_2(s)q_1(s)$, $f_2(s) = \overline{T}_1(s)q_2(s)$.

Going over from (26) to originals, we obtain

$$a=\frac{\tilde{F}_1^2(\tau)}{\tilde{F}_2^2(\tau)}, \tag{27}$$

where

$$\tilde{F}_1(\tau)=\frac{R_0}{2\sqrt{\pi}}\int_0^\tau f_2(\tau-\theta)\int_0^\theta(\theta-\vartheta)^{-1/2}[f_2(\vartheta)-f_1(\vartheta)]\,d\vartheta d\theta; \tag{27'}$$

$$\tilde{F}_2(\tau)=\int_0^\tau(2\theta-\tau)f_1(\tau-\theta)f_2(\theta)\,d\theta; \tag{27''}$$

$$f_1(\theta)=\int_0^\theta q_1(\vartheta)\overline{T}_2(\theta-\vartheta)\,d\vartheta,\quad f_2(\theta)=\int_0^\theta q_2(\vartheta)\overline{T}_1(\theta-\vartheta)\,d\vartheta. \tag{27'''}$$

In the case when the source density has a normal distribution over a surface, by starting from the relationship (7) we have

$$a\frac{d}{ds}[\overline{T}(s)/q_0(s)]=\frac{1}{4\gamma}\frac{\overline{T}(s)}{q_0(s)}-\frac{1}{4\gamma}\frac{1}{\sqrt{\lambda c\rho}\sqrt{s}},\quad \overline{T}(s)=T(0,\ 0,\ s), \tag{28}$$

from which

$$a[\overline{T}'(s)q_0(s)-\overline{T}(s)q_0'(s)]=\frac{1}{4\gamma}\overline{T}(s)q_0(s)-\frac{1}{4\gamma\sqrt{\lambda c\rho}}s^{-1/2}q_0^2(s) \tag{29}$$

or in the space of originals

$$\gamma a\tilde{\tilde{\varphi}}_1(\tau)+(\lambda c\rho)^{-1/2}\tilde{\tilde{\varphi}}_2(\tau)=\tilde{\tilde{\varphi}}_3(\tau), \tag{30}$$

where

$$\tilde{\tilde{\varphi}}_1(\tau)=\int_0^\tau(2\theta-\tau)\overline{T}(\tau-\theta)q_0(\theta)\,d\theta; \tag{30'}$$

$$\tilde{\tilde{\varphi}}_2(\tau)=\frac{1}{4\sqrt{\pi}}\int_0^\tau q_0(\tau-\theta)\int_0^\theta q_0(\vartheta)(\theta-\vartheta)^{-1/2}d\vartheta d\theta; \tag{30''}$$

$$\tilde{\tilde{\varphi}}_3(\tau)=\frac{1}{4}\int_0^\tau\overline{T}(\tau-\theta)q_0(\theta)\,d\theta. \tag{30'''}$$

The functional relationship (30) can be used as in the relationship (19) above to find the parameters a, $\lambda c\rho$ by using measurement data in one or in two realizations with different $q_0(\tau)$, γ.

By using the superposition principle and taking account of the dependence (4), a functional relation is easily set up between a, λ, T, Q for a set of actions N of concentrically arranged sources concentrated on circles $R = nR_0$, $n = 1$, N as $N \to \infty$. In this case we have

$$\frac{\overline{T}(s)}{Q_0(s)}=\frac{1}{2\lambda}\left[\operatorname{cth}\left(\sqrt{\frac{s}{a}}\frac{R_0}{2}\right)-1\right],\quad Q_0(s)=\frac{Q(s)}{2\pi R_0}\equiv Q_n(s), \tag{31}$$

from which

$$\frac{\sqrt{a}}{R_0}\left[\frac{\overline{T}(s)}{Q_0(s)}\right]'+\frac{\lambda}{2}\left[\frac{\overline{T}(s)}{Q_0(s)}\right]^2\frac{1}{\sqrt{s}}+\frac{1}{2\sqrt{s}}\frac{\overline{T}(s)}{Q_0(s)}=0, \tag{32}$$

which corresponds to

$$\frac{\sqrt{a}}{R_0}[\bar{T}'(s)\,Q_0(s)-\bar{T}(s)\,Q_0'(s)]+\frac{\lambda}{2}\bar{T}^2(s)\,s^{-1/2}+ \\ +\frac{1}{2}s^{-1/2}\bar{T}(s)\,Q_0(s)=0 \tag{33}$$

or in the space of originals

$$\frac{\sqrt{a}}{R_0}\varphi_1^*(\tau)+\frac{\lambda}{2}\varphi_2^*(\tau)+\varphi_3^*(\tau)=0, \tag{34}$$

where

$$\varphi_1^*(\tau)=\int_0^\tau(2\theta-\tau)\,\bar{T}(\tau-\theta)\,Q_0(\theta)\,d\theta; \tag{34'}$$

$$\varphi_2^*(\tau)=\int_0^\tau\bar{T}(\tau-\theta)\int_0^\theta\pi^{-1/2}(\theta-\vartheta)^{-1/2}\bar{T}(\vartheta)\,d\vartheta d\theta; \tag{34''}$$

$$\varphi_3^*(\tau)=\frac{1}{2}\int_0^\tau\bar{T}(\tau-\theta)\int_0^\theta\pi^{-1/2}(\theta-\vartheta)^{-1/2}Q_0(\vartheta)\,d\vartheta d\theta. \tag{34'''}$$

Formula (34) can be used to determine the parameters a, λ by involving results of measuring T, q in one or two realizations analogously to what was done in examining the dependence (19).

On the basis of the dependences obtained, algorithms were developed to compute a set of TPC on an electronic computer for different kinds of local heat sources. Checking the algorithms developed was performed by formulating a number of numerical experiments where results of solving appropriate direct problems of nonstationary heat conductivity were used as initial data.

Results of estimates of the parameters a, $\lambda c\rho$ in application to a version of specimen heating by a local source distributed uniformly within the limits of a circle of radius 0.005 m are represented in Fig. 1, where the following values of the material TPC were taken: $a = 1\cdot10^{-5}$ $m^2\cdot sec^{-1}$; $\lambda c\rho$ = 129.6 kW$\cdot$ kJ$\cdot m^{-4}\cdot K^{-2}$; λ = 0.036 kW$\cdot m^{-1}\cdot K^{-1}$. Estimates of the parameters a, $\lambda c\rho$ represented in Fig. 1, refer to the case of using "measurement"

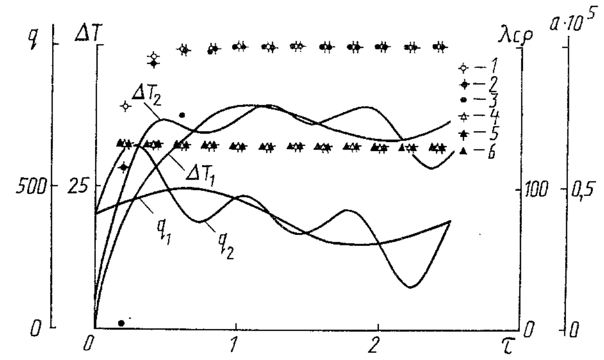

Fig. 1. Initial data for the computation and results of estimates of the parameters a, $\lambda c\rho$ during specimen heating by a heat source distributed uniformly within the limits of a circle of radius 0.005 m: ΔT_1, q_1 are the specimen surface temperatures at the source center and the thermal flux density from a source in the first realization; ΔT_2, q_2 is the same as above in the second realization: 1, 4) are the parameters a, $\lambda c\rho$, computed according to ΔT, q of the first realization; 2, 5) are the same over ΔT, q of the second realization; 3, 6) are the parameters a, $\lambda c\rho$, computed by using the data of "measurements" of two realizations. ΔT, K; q, kW$\cdot m^{-2}$; a, $m^2\cdot sec^{-1}$; $\lambda c\rho$, kW$\cdot$kJ$\cdot m^{-4}\cdot K^{-2}$; τ, c.

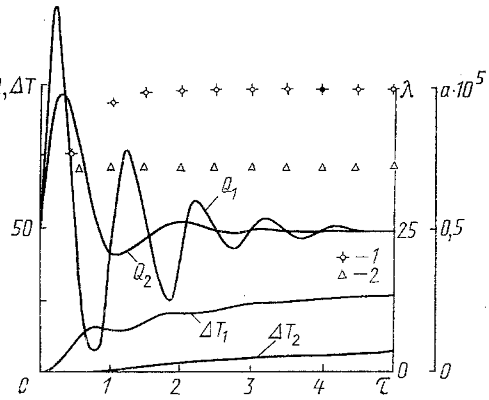

Fig. 2. Initial data for the computation and results of estimates of the parameters a, λ during specimen heating by local heat sources concentrated on circles of radii 0.005 and 0.01 m: Q_1, ΔT_1 are the power of the source and the temperature at the center of the heating zone in a realization with source radius 0.005 m; Q_2, ΔT_2 is the same in a realization with source radius 0.01 m; 1 are estimates of the parameter a; 2 are estimates of the parameter λ. Q, W; λ, $W \cdot m^{-1} \cdot K^{-4}$.

data for T, q in both some one realization and in two realizations with different heating conditions. As is seen from the results presented in Fig. 1, the computed values of the parameters a, $\lambda c\rho$ converge sufficiently rapidly to their exact values as the duration of the realization time intervals used for the processing increases.

Results of the estimates of the parameters a, λ in application to the case of using a set of "measurement" data for T, Q in realizations with sources concentrated on circles of radii 0.005 and 0.01 m are represented in Fig. 2. The thermophysical characteristics of the specimen material are taken the same as in the case considered above. The results of testing the developed numerical algorithms represented in Fig. 2 indicate the possibility of efficient restoration of the TPC by using the proposed approach. This latter is verified by the sufficiently rapid convergence of the computed values of a, λ to the true values as a function of the duration of the realization time intervals utilized to calculate the functions entering the computed dependences.

Analogous results hold also for other kinds of sources examined in this paper.

Therefore, on the basis of the analysis performed, a deduction can be made that the exact explicit dependences obtained that relate the specimen TPC to the results of measuring different parameters under power varying arbitrarily in time for local heat sources of different kinds, can be the basis of practical method of determining the TPC of materials.

Notation. T is the temperature; Q is the heat source power; q is the heat flux; r, z are spatial coordinates; τ, θ, ϑ are times; a, is thermal diffusivity; λ is thermal conductivity; c is specific heat; ρ is density and J_ν is the Bessel function of order ν.

LITERATURE CITED

1. A. G. Shashkov, V. P. Kozolov, and V. N. Kipovtsev, Inzh.-fiz. Zh., 50, No. 3, 456-465 (1986).
2. V. P. Kozlov, Inzh.-fiz. Zh., 50, No. 4, 659-666 (1986).
3. A. G. Shashkov, V. P. Kozlov, and A. V. Stankevich, Inzh.-fiz. Zh., 50, No. 2, 303-309 (1986).
4. G. T. Aldoshin, A. S. Golosov, V. I. Zhuk, et al., Inzh.-fiz. Zh., 55, No. 6, 989-997 (1988).
5. A. P. Purdnikov, Yu. A. Brychkov, and O. M. Marichev, Integrals and Series. Special Functions [in Russian], Moscow (1983).
6. A. V. Lykov, Theory of Heat Conduction [in Russian], Moscow (1967).